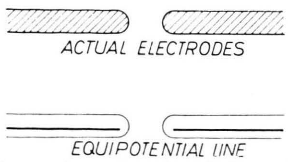

Fig. 7. Comparison of real and theoretical cases.

These two facts imply that the following series expansions are valid:

$$a = s_1 \sum a_{klpq}\, v_1{}^k\, v_2{}^l\, v_3{}^p\, v_4{}^q$$

and analoguous for b, c, d, and e. In § 1 the first terms of these expansions have been found, and the expansion for b gets the more special form

$$b = \tfrac{1}{2}\, s_1\, v_1 \sum b_{klpq}\, v_1{}^k\, v_2{}^l\, v_3{}^p\, v_4{}^q$$

and analoguous for c, d, and e.

By substituting the series expansions of the parameters a, b, c, d, and e with only first order terms in v_n included $(k+l+p+q=1)$ in the formulae (3), it is easily proved, that these first order terms are all zero.

Now we suppose, that we have the series expansions up to the $(2\,n+1)^{\text{st}}$ order:

$$a = \tfrac{1}{2}\, s_1 (1 + A_{2n} + a_{2n}) ,$$

$$b = \tfrac{1}{2}\, s_1\, v_1 (1 + B_{2n} + b_{2n}) \quad \text{etc.}$$

where A_{2n}, B_{2n}, etc. contain only even powers of the v_k up to the order $2\,n$ and a_{2n} or b_{2n} etc. is the term of lowest order, having odd powers of one or more of the v_k, this order thus being $2\,n$ or $2\,n+1$. Evaluating

$$a^2, \quad a^2 - b^2, \quad a^2/b^2, \quad r_1 (b/a)^{2m+1}, \quad \text{etc.}$$

which are in fact the only expressions, occurring in the equations (3) after performing the series expansions for the logarithms, one will find that odd terms of the order $2\,n$ or $2\,n+1$ only enter as homogeneous first order expressions in a_{2n}, b_{2n} etc. Equating all terms of this order to zero, provides:

$$a_{2n} = b_{2n} = \ldots = 0 .$$

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Shape of the Magnetic Field between Conical Pole Faces

By A. J. H. BOERBOOM and H. A. TASMAN

Laboratorium voor Massaspectrografie, Amsterdam, The Netherlands

and H. WACHSMUTH

Physikalisches Institut der Technischen Hochschule, München

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Calculation is made of the shape of the magnetic field between conical pole faces, which may be used as an inhomogeneous deflecting field for a mass spectrometer. The results are expressed as a series expansion in the coordinates around the main path, and in the gap width at the radius of the main path.

The application of ψ-independent, inhomogeneous magnetic deflecting fields for mass spectrometers offers the possibility of considerable increase in resolving power without increase of radius or decrease of slit widths (TASMAN and BOERBOOM [1]; WACHSMUTH, BOERBOOM and TASMAN [2]; TASMAN, BOERBOOM and WACHSMUTH [3]). Hereto magnetic fields are required, which decrease with increasing radius. The simplest way to create such fields is the use of conical pole faces, between which the gap increases with increasing radius. The present calculations provide the shape of the resulting field, as a power expansion in the normal and binormal coordinates, and the gap width at the radius r_{m} of the main path, for the symmetrical case with respect to the median plane.

The coordinate system is shown in Fig. 1; a radial section is represented in Fig. 2. Use is made of the dimensionless coordinates: normal coordinate $u = (r - r_{\mathrm{m}})/r_{\mathrm{m}}$; binormal coordinate $v = z/r_{\mathrm{m}}$; path coordinate $w = \psi$. The gap width at $u = 0$ equals $2\,b\,r_{\mathrm{m}}$.

1. The scalar magnetic potential

The scalar magnetic potential φ_{m} is related to the magnetic field strength $\mathfrak{B}$ through its definition:

[1] H. A. TASMAN and A. J. H. BOERBOOM, Z. Naturforschg. **14 a**, 121 [1959].

[2] H. WACHSMUTH, A. J. H. BOERBOOM and H. A. TASMAN, Z. Naturforschg. **14 a**, 818 [1959].

[3] H. A. TASMAN, A. J. H. BOERBOOM and H. WACHSMUTH, Z. Naturforschg. **14 a**, 822 [1959].

$$\mathfrak{B} = -\operatorname{grad} \varphi_{\mathrm{m}} . \tag{1}$$

The field shape is supposed to be independent of w. The scalar value of the field strength at the main path ($u = v = 0$) is designated by B. The scalar mag-

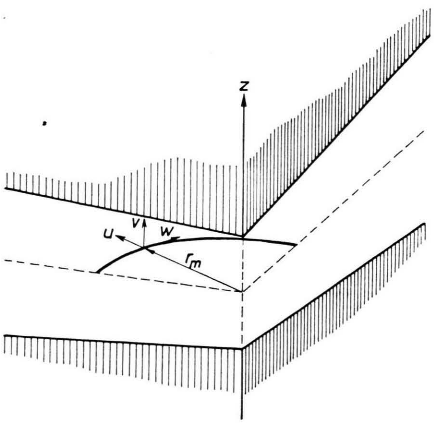

Fig. 1. Coordinate system.

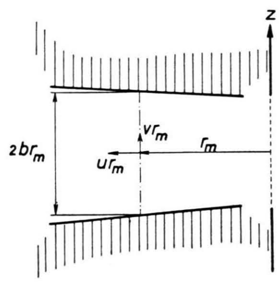

Fig. 2. Radial section.

netic potential is anti-symmetrical with respect to the median plane $v = 0$. We expand φ_{m}/B in a power series in u and v:

$$\varphi_{\mathrm{m}}/B = \sum_{k,\, l=0}^{\infty} a_{k,\, l}\, u^k\, v^l , \tag{2}$$

where the symmetry causes the coefficients $a_{k,\, l}$ to vanish for $l =$ even.

Now φ_{m}/B obeys the LAPLACIAN equation:

$$\nabla^2 \varphi_{\mathrm{m}}/B = 0 . \tag{3}$$

In our case, the operator ∇^2 reads:

$$\nabla^2 = \frac{1}{r_{\mathrm{m}}^2} \left\{ \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} + \frac{1}{(1+u)} \frac{\partial}{\partial u} \right\} . \tag{4}$$

On insertion of (2) into (3) – (4), we obtain on equating the coefficient of the $u^{k+1}\, v^l$-term to zero:

$$\begin{aligned} &(l+2)\,(l+1)\,(a_{k,\, l+2} + a_{k+1,\, l+2}) \\ &\quad = -\,(k+3)\,(k+2)\, a_{k+3,\, l} - (k+2)^2\, a_{k+2,\, l} . \end{aligned} \tag{5}$$

We choose as independent variables the coefficients $a_{k,\, 1}$, which determine all other coefficients through (5), together with the condition:

$$a_{k,\, l} = 0 \text{ for } k < 0 . \tag{6}$$

On writing out the expressions for a_{03}, a_{13}, a_{23}, etc., and a_{05}, a_{15}, a_{25}, etc., and so on, expressed in the independent variables $a_{k,\, 1}$, one arrives at the general expression:

$$\begin{aligned} &(l+1)\,(l+2)\, a_{k,\, l+2} \\ &\quad = (-1)^k \sum_{i=1}^{k+1} \{(-1)^i\, i\, a_{i,\, l}\} - (k+1)\,(k+2)\, a_{k+2,\, l} . \end{aligned} \tag{7}$$

2. Conical pole faces

A conical pole face may be represented by:

$$v = a\, u + b , \tag{8}$$

which should be an equipotential surface for the scalar magnetic potential φ_{m}. We assume $a = p\, b$, and expand the coefficients $a_{k,\, l}$ in (2) in a power series in b:

$$\varphi_{\mathrm{m}}/B = \sum_{k,\, l,\, m=0}^{\infty} a_{k,\, l,\, m}\, u^k\, v^l\, b^m , \tag{9}$$

where $l =$ odd and $m =$ even, because of the symmetry. The coefficients $a_{k,\, l,\, m}$ are now independent of b.

The scalar magnetic potential at the conical pole face should be independent of u. Substituting

$$v = (p\, u + 1)\, b \tag{10}$$

into (9), the sum of the terms with $k \neq 0$ should be zero. Of this group, the sum of the terms with $l + m = 1$; $l + m = 3$; etc., and any specified value of k, should be individually zero, because the coefficients $a_{k,\, l,\, m}$ do not depend on b.

The sum of the terms with $l + m = 1$ leads to the equality:

$$a_{k,\, 1,\, 0} + p\, a_{k-1,\, 1,\, 0} = 0 . \tag{11}$$

In accordance with the convention in the previous article [1], we choose:

$$a_{0,\, 1,\, 0} = -1 , \quad a_{0,\, 1,\, m} = 0 \text{ for } m \neq 0 \tag{12}$$

which leads to:

$$\left. \begin{aligned} &a_{1,\, 1,\, 0} = p , \quad a_{2,\, 1,\, 0} = -p^2, \\ &a_{3,\, 1,\, 0} = p^3, \quad a_{k,\, 1,\, 0} = (-1)^{k+1}\, p^k . \end{aligned} \right\} \tag{13}$$

The sum of the terms with $l+m=3$ equals:

$$a_{k,1,2}+p\,a_{k-1,1,2}+a_{k,3,0}+3\,p\,a_{k-1,3,0} + 3\,p^2\,a_{k-2,3,0}+p^3\,a_{k-3,3,0}=0\,. \tag{14}$$

From (7), it follows that:

$$(l+1)\,(l+2)\,a_{k,l+2,0} = (-1)^k \sum_{i=1}^{k+1} \{(-1)^i\,i\,a_{i,l,0}\} - (k+1)\,(k+2)\,a_{k+2,l,0}\,. \tag{15}$$

On expressing the coefficients with $l=3$ in (14) in coefficients with $l=1$ by means of (15), we arrive at the equality:

$$a_{k,1,2}+p\,a_{k-1,1,2}=(-1)^k\,\frac{p-p^2}{6} \tag{16}$$

and because of (12):

$$a_{k,1,2}=(-1)^k\,\frac{p-p^{k+1}}{6}\,. \tag{17}$$

The v-component of the field strength in the median plane may be found from (1), and a summation of (13) and (17):

$$B_v=B\left[\frac{1}{1+p\,u}+\frac{b^2}{6}\left\{\frac{p\,u}{1+u}-\frac{p^2\,u}{1+p\,u}\right\}+\dots\right]. \tag{18}$$

The parameters used in the previous article [1] are related to the coefficients $a_{k,l}$ in (2) through:

$$a_{01}=-1\,; \qquad a_{11}=n\,; \tag{19}$$
$$a_{21}=\tfrac{1}{2}\{X(1-n)-2\,n\}\,; \quad a_{31}=-C_3\,; \; a_{41}=-C_4\,.$$

By reversion of the series one finds the required cone angle for a prescribed n to be defined through:

$$p=n+\frac{1}{6}\,n(1-n)\,b^2+\dots, \tag{20}$$

whereas the resulting field shape is represented by:

$$X=2\,n+\frac{1}{3}\,n(1-n)\,b^2+\dots,$$
$$C_3=-n^3+\frac{1}{6}\,n(1-n)^2\,(1+2\,n)\,b^2+\dots, \tag{21}$$
$$C_4=n^4-\frac{1}{6}\,n(1-n)^2(1+2\,n+3\,n^2)b^2+\dots\,.$$

The omitted terms in (20) and (21) are of the order b^4.

The results of this work were obtained independently by the three authors by different methods. The derivation of one of us (A. J. H. B.) was presented in this paper.

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Calculation of the Ion Optical Properties of Inhomogeneous Magnetic Sector Fields, Part 2: The Second Order Aberrations Outside the Median Plane

By H. WACHSMUTH,

Physikalisches Institut der Technischen Hochschule, München,

and A. J. H. BOERBOOM and H. A. TASMAN,

F.O.M.-Laboratorium voor Massaspectrografie, Amsterdam, Holland

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In a previous article [1] we pointed out the possible advantages of inhomogeneous magnetic sector fields for mass spectrometers, as these fields permit a substantial increase in mass dispersion and resolving power without change in radius or slit widths. In the said paper [1] we calculated the coefficients of the second order aberrations in the median plane, as well as the field shape required to eliminate the second order angular aberration in the median plane. In the present paper we calculate the second order aberrations outside the median plane referring to focusing in the radial direction. Again the influence of fringing fields is being neglected, and the field boundaries are supposed to be plane and normal to the main path at the point where it enters and leaves the field.

The use of an inhomogeneous magnetic analysing field for a mass spectrometer may result in a greatly enlarged mass dispersion and resolving power without change in radius or slit widths. We discussed

[1] H. A. TASMAN and A. J. H. BOERBOOM, Z. Naturforschg. 14 a, 121 [1959].